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Letters

Computation of the Hecken Impedance Function

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The Dolph–Chebyshev impedance function derived by Klopfenstein [1] has discontinuities at the taper ends which introduce unwanted effects in certain applications. The Hecken impedance function [2] is not optimum in the Dolph–Chebyshev sense, but achieves matching without impedance steps. For any bandwidth and maximum magnitude of reflection coefficient in the passband, the Hecken taper is only slightly longer than the optimum taper [2]. Hecken's near-optimum taper is therefore an attractive alternative to the optimum taper when impedance discontinuities are undesirable.

The equation for the near-optimum impedance function contains a transcendental function $G(B,\xi)$ which is tabulated in Hecken's paper. The function is given by

$$G(B,\xi) = \frac{B}{\sinh B}\int_0^{\xi} I_0\{B\sqrt{1-\xi'^2}\}\, d\xi'$$

where $I_0(z)$ is the modified Bessel function of the first kind and zero order.

Instead of using the tables, $G(B,\xi)$ may be computed recursively as

$$G(B,\xi) = \frac{B}{\sinh B}\sum_{k=0}^{\infty} a_k b_k$$

where

$$a_0 = 1 \quad a_k = \frac{B^2}{4k^2}a_{k-1}$$

$$b_0 = \xi \quad b_k = \frac{\xi(1-\xi^2)^k + 2kb_{k-1}}{2k+1}.$$

The derivation is based on the method described by Grossberg [3] and is not given here.

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Synthesis of Certain Transmission Lines Employed in Microwave Integrated Circuits

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With a quasi-TEM approximation, the characteristic parameters of numerous structures used as hyperfrequency microelectronics transmission lines can be calculated with the aid of conformal mapping. Simple theoretical formulas are rarely used since they bring into play the function $K(k)/K'(k)$ where $K(k)$ is the complete elliptic integral of the first type, $K'(k)$ its complementary function, and k its argument.

Some geometrical configurations which can be treated are shown in Fig. 1(a)–(c). This method is particularly interesting since expressions of k (argument of elliptic integral) as a function of geometric dimensions are often simple.

The infinite dielectric thickness hypothesis made in certain cases is, in general, justified by the spacing between conductors. Although this method is surprisingly simple accompanied by a large application domain, it has been put aside by many research workers. Instead, sophisticated numerical methods like those of finite differences and finite elements [1] have been preferred. These methods are applicable for the analysis of transmission lines but not for the synthesis. Moreover, they do not lead to

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